Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 28/11/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Determine the radius of convergence of e^x. (02)

b. Show that $P_n(x) = (-1)^n P_n(-x)$, where $n \in \mathbb{N} \cup \{0\}$. (02)

c. $\int_{-1}^{1} J_1(x)\ dx =$ _____________. (01)

d. Find the singular points of the differential equation $x(x-1)y'' + x^2y' + y = 0$. (01)

e. State orthogonal property of Bessel's function. (01)

Q-2 **Attempt all questions** **(14)**

a. Find the power series solution in powers of $(x-1)$ of the initial value problem (05)

$$xy'' + y' + 2y = 0,\ y(1) = 1,\ y'(1) = 2.$$

b. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$. (05)

c. Express $Q(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomials. (04)

OR

Q-2 **Attempt all questions** **(14)**

a. Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)\, t^n$. (05)

b. Obtain power series solution of the differential equation $y'' + xy' + y = 0$ about $x = 0$. (05)

c. Show that infinity is not a regular singular point of the Bessel equation (04)

$$x^2y'' + xy' + (x^2 - n^2)y = 0.$$

Q-3 **Attempt all questions** **(14)**

a. Prove that (06)

(1) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x,$

(2) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x.$

b. Expand $f(x)$ in the form $\sum_{n=0}^{\infty} c_n P_n(x)$, where $f(x) = \begin{cases} 0, -1 < x < 0 \\ 1, \ \ 0 < x < 1 \end{cases}$. (05)

c. Find a recurrence formula for the power series solution around $x = 0$ for the differential equation $y'' - xy' + 2y = 0$. (03)

OR

Q-3 Attempt all questions (14)

a. Show that $\int_{-1}^{1} P_n(x)\,(1 - 2tx + t^2)^{-\frac{1}{2}}\,dx = \frac{2t^n}{2n+1}$, when $|t| < 1$ and $|x| \le 1$. (06)

b. Use the method of Frobenius to find one solution near $x = 0$ of (05)

$$x^2 y'' - xy' + y = 0.$$

c. When n is a nonnegative integer, show that $\frac{d^n}{dx^n} J_n(x)\Big|_{x=0} = \frac{1}{2^n}$. (03)

SECTION – II

Q-4 Attempt the following questions (07)

a. Show that the partial differential equations $p = 6x - 4y + 1$, $q = 4x + 6y + 1$ do not have any common solution. (02)

b. Eliminate a and b from $z = axe^y + \left(\frac{1}{2}\right) a^2 e^{2y} + b$. (02)

c. Define: Semi-linear equation. (01)

d. Find a complete integral of $z = px + qy + \sin(p + q)$. (01)

e. Define: Pochhammer symbol. (01)

Q-5 Attempt all questions (14)

a. Prove that if $\vec{X}$ is a vector such that $\vec{X} \cdot curl\, \vec{X} = 0$ and μ is an arbitrary function of x, y, z, then $\mu\vec{X} \cdot curl\, \mu\vec{X} = 0$. (05)

b. Prove that (05)

(1) $\log(1 - x) = -x\, F(1,1;2;x)$,

(2) $\lim_{a\to\infty} F\left(1, a; 1; \frac{x}{a}\right) = e^x$.

c. Show that the equations $f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$. (04)

OR

Q-5 Attempt all questions (14)

a. Prove that a necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x and y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$. (05)

b. Show that $\frac{d}{dx} F(\alpha, \beta; \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \gamma + 1; x)$ and deduce that (05)

$\frac{d^n}{dx^n} F(\alpha, \beta; \gamma; x) = \frac{(\alpha)_n (\beta)_n}{(\gamma)_n} F(\alpha + n, \beta + n; \gamma + n; x)$.

c. Solve: $p - 2q = 3x^2 \sin(y + 2x)$. (04)

Q-6 Attempt all questions (14)

a. Find a complete integral of the equation $(p^2 + q^2)y = qz$ by Charpit's method. (06)

b. Verify that the Pfaffian differential equation (05)

$$y(1 + z^2)dx + x(1 + z^2)dy + xydz = 0$$

isintegrable and find its solution.

c. Form a partial differential equation by eliminating the function f from (03)

$f(z + xy, x^2 + y^2) = 0$.

OR

Q-6 **Attempt all questions** **(14)**

a. Find a complete integral of $z^3 = pqxy$ by Jacobi's method. (06)

b. Find the third approximation of the solution of the equation (05)

$$\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$$

by Picard's method.

c. Solve: $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z.$ (03)